

Critical mass of neutron stars : a new view

Parthasarathi Majumdar*

A P C Division

Saha Institute of Nuclear Physics

Kolkata 700064, India.

March 3, 2013

Abstract

The issue of the critical mass of neutron stars, with respect to gravitational collapse to black holes, is reexamined from the perspective of thermal stability of quantum horizons. Postulating the existence of a tiny, *embryonic*, isolated horizon, hidden deep inside a gravitationally contracting neutron star, the critical mass is seen to emerge from the extrapolation of the criterion of thermal stability of quantum isolated horizons derived earlier by us, to such a ‘hidden’ horizon, as a condition of its stability and growth (through formation of *trapping* or *dynamical* horizons), eventually leading to an equilibrium isolated horizon engulfing the entire star. The perspective is based on aspects of Loop Quantum Gravity, and in contrast to extant analyses in the neutron star literature, uses neither classical spacetime metrics nor details of strong nucleonic interactions at supranuclear densities, thus delineating the essential role of quantum gravitation in black hole formation.

1 Introduction

The observed absence of neutron stars with masses in large excess of a few solar masses strongly suggests the existence of a critical maximum mass for such stars. Heavier neutron stars must become *unstable* with respect to gravitational collapse and metamorphose into black holes. Since indirect evidence for black hole candidates with masses ranging from a few to more than a billion solar masses have been widely reported, the importance of understanding the critical mass of neutron stars can hardly be overemphasized. Indeed, this has been an issue of some interest for over three decades in neutron star research [1, 2, 3, 4, 5, 6, 7, 8]. Most approaches, with the exception of [8] are variations on the theme of derivation of the Chandrasekhar bound on critical masses of dying stars [9] for which they may escape the white dwarf stage. This standpoint entails hydrostatic equilibrium between the gravitational pressure at the centre of the neutron star, on the one hand, and Fermi degeneracy pressure due to the neutron star core modelled as a degenerate relativistic neutron gas, on the other. This is invoked to determine the highest critical mass that a neutron star can possess before it collapses to a black hole due to gravitational predomination. More contemporary variations focus on details of the equation of state of dense neutron (or quark) matter as appropriate to the inner regions of a neutron star. Since the density in these regions exceeds nuclear densities ($\sim 10^{14}$ gm/cc), one is compelled to model the strong interaction between nucleons and also thresholds of production of hyperons and other resonances, through low energy effective field theory models of strong interaction dynamics at high density. The equation of state to be used to ascertain the Fermi degeneracy pressure is therefore rather sensitive to the details of the low energy effective model

*parthasarathi.majumdar@saha.ac.in

employed, and therefore susceptible to the ambiguities inherent in its extraction from QCD, for instance. Perhaps such ambiguities are unavoidable to some extent at the present state of development of low energy strong interaction physics, if a sufficiently detailed knowledge of the equation of state of dense nucleonic matter is desired. One could question though to what extent such details are actually necessary for the estimate of a stability upper bound on the mass of a neutron star against gravitational collapse.

Furthermore, observe that the low energy effective strong interaction models employed to determine the equation of state in the inner regions of a neutron star are all formulated in a *flat Minkowski* spacetime background. In contrast, the gravitational pressure causing collapse is computed within *general relativity* using the Tolman-Oppenheimer-Volkoff equation. When considering hydrostatic equilibrium, one might worry that this obvious inconsistency in the approach may adversely affect the final result. In other words, consistency demands that models to determine the equation of state be formulated in accord with principles of *general relativistic* quantum field theory. This challenging task has not really been taken up in earnest as far as I know. Under the circumstances, one may fear that even the best Minkowski space determination of the equation of state may continue to suffer from large errors due to neglect of spacetime curvature deep inside a neutron star. It is quite remarkable however that such large errors are apparently not there. This situation is reminiscent of John Mitchell's derivation of the Schwarzschild radius of a gravitating sphere more than a century before Schwarzschild's own derivation, or more prominently, Niels Bohr's derivation of the Bohr radius of the hydrogen atom, longer than a decade prior to the advent of quantum mechanics, or even Hans Bethe's estimate of the Lamb Shift in hydrogen prior to the full Lorentz-covariant quantum electrodynamics calculation by French and Weiskopff and Feynman. In each of these examples, though, the antiquated methodology is supplemented by some novel physical insights which point to the correct theory developed in due course. It is an interesting question as to which theory Chandrasekhar's approach is a pointer to.

In any event, a systematic approach to the problem of the maximum mass of a stable neutron star should involve *general relativistic* quantum field theory rather than special relativistic, and this is well-known to be a challenging problem even in its formulation. One of the key problems of quantum field theory in curved spacetime is that standard results of special relativistic quantum field theory, like the spin statistics theorem, which we take for granted in flat spacetime, need to be established anew, that too on a case-by-case basis, depending on the background geometry [10].

We have already mentioned above that ref. [8] presents an exception to the standpoint of most extant approaches. In this work, a maximum mass of the neutron star has been derived based on the gravitational hydrostatic pressure obtained from the Tolman-Oppenheimer-Volkoff equation, the requirement that there is no local spontaneous collapse (Le Chatelier's Principle) and the special relativistic requirement that the sound velocity is less than the speed of light in vacuum. Thus, this approach is insensitive to the details of the microscopic model of strong nucleonic interactions employed to determine the equation of state of dense neutron matter. However, it does crucially use the spherically symmetric ansatz of the classical spacetime metric in the Tolman-Oppenheimer-Volkoff equation.

In this paper we offer an alternative perspective, which is not only quite insensitive to details of the strong interaction dynamics at high density, but in fact also does not depend on the details of the classical spacetime geometry (metric) inside the star. Thus, the paper may be considered to be an attempt to go beyond ref. [8]. We begin with the observation that the stability upper bound for a neutron star can be expressed as an *instability lower* bound, i.e., the minimum mass of a neutron star for which it will be unstable to gravitational collapse to a black hole

$$\frac{M_{crit}}{M_P} > \xi \left(\frac{\lambda_{Cn}}{l_P} \right)^2 = \xi \frac{A_{Cn}}{A_P} \quad (1.0.1)$$

where, M_P , l_P , A_P are respectively the Planck mass, length and of the area, ξ is a dimensionless constant of $O(1)$ and λ_{Cn} is the Compton wavelength of the neutron, with $A_{Cn} = \lambda_{Cn}^2$. Clearly, the right hand side of (1.0.1) is *non-perturbative* in the Planck length l_P , as distinct from perturbative quantum

gravity effects which are $O(l_P^n)$, $n \in \mathbf{Z}$. In fact, it is reminiscent of black hole entropy [11, 12]

$$S_{bh} = \frac{A_{hor}}{4A_P} + \text{quantum corrections} \quad (1.0.2)$$

which, likewise, arises from non-perturbative quantum spacetime fluctuations. The question that arises then is : *Is the minimum mass of a neutron star, for which it becomes unstable with respect to gravitational collapse to a black hole, related in any way to the entropy of the black hole ?* Unlikely as it may seem, the answer to this question will turn out to be in the affirmative, of course within a specific approach to quantum gravity, namely Loop Quantum Gravity (LQG). The actual value of the critical mass that emerges from this relationship lies within the correct order of magnitude, even though there is an ambiguity of $O(1)$ which our approach cannot fix at the present state of development.

The paper is organized as follows: in the next section, we outline our proposal of formation of an isolated horizon from a collapsing neutron star, starting with a hidden, ‘embryonic’ horizon (whose spatial foliation is an outer trapped 2-surface). This is followed in section 3, by a digression into a sketch of our earlier derivation of a thermal stability criterion of an equilibrium isolated horizon within a canonical or a grand canonical ensemble of thermally active trapping horizons, based on the notion of *Thermal Holography*. Even though most of the material in this section is already published, its inclusion here is primarily to make the paper self-contained. The stability criterion derived here, expressed as an inequality between the mass of the isolated horizon and its *microcanonical* entropy (in Planckian) units, is the key to our derivation of the critical mass of a neutron star, as shown in section 4. We conclude in section 5 with a discussion of our result and future outlook.

2 Horizon formation

We begin with the postulate that the formation of a horizon (a hypersurface in spacetime whose spatial foliation is an outer trapped 2-surface) is not an abrupt event, but rather an endpoint of a continuous succession of events, beginning with the formation of a tiny, hidden (isolated) horizon deep inside the collapsing neutron star and its subsequent growth. In this sense, the eventual formation of an event or isolated horizon from a collapsing neutron star is analogous to a first order phase transition proceeding by bubble nucleation. The incipient ‘embryonic’ horizon forms by a process presumably arising from quantum spacetime fluctuations through a mechanism that is not yet known. Once the hidden baby horizon has formed, its stability (and future growth) satisfies a certain thermal stability criterion (to be elaborated upon in sections 3 and 4) which has been derived [13, 14, 15] on a far sounder basis, from various aspects of loop quantum gravity [16]. This stability criterion is expressed as a lower bound on the mass of the hidden horizon in terms of its cross-sectional area, in Planckian units.

A pictorial representation is shown in Figs 1-3.

In Fig.1, on the left panel a cartoon is shown of a spherically collapsing star in Eddington-Finkelstein coordinates; the broken ellipses depict the collapsing star and the vertical unbroken lines are the event horizons formed when the star collapses down to a certain size. The collapse continues beyond the formation of the horizon, all the way to a singularity, shown here by the broken vertical line.

The right panel of Fig. 1 depicts a picture of gradual growth of the proposed nascent hidden horizon into the physical horizon which coincides with the standard conceptualization. The filled ellipses inscribed inside the collapsing star represented by the broken ellipses indicate this growth. Since, eventually the entire star is now eaten up by the growing horizon, there is no more energy to be accreted from the star ‘outside’, and the growth stops. Subsequent to this, the star continues to collapse, eventually reaching the black hole singularity.

Because of the teleological nature of event horizons of black holes where a global timelike isometry is

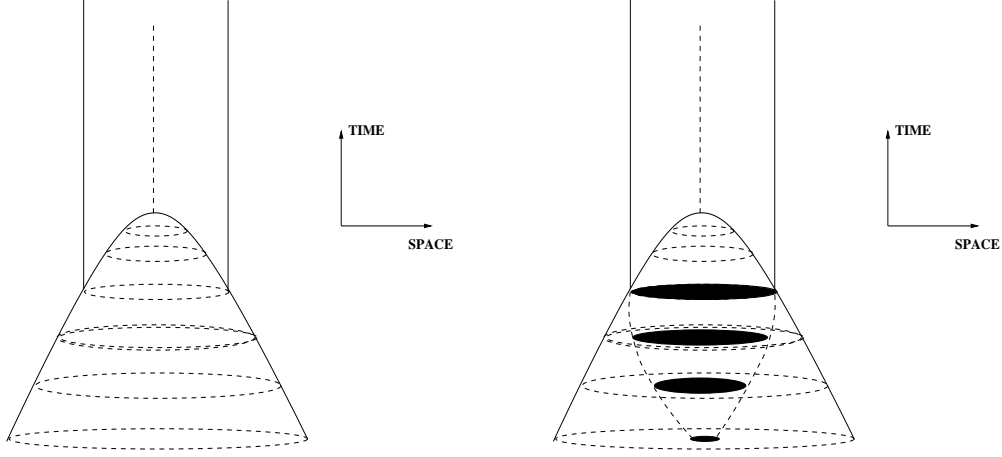


Figure 1: *Left panel:* Gravitational collapse of a spherical star in Eddington-Finkelstein coordinates. *Right panel:* Growth of nascent hidden horizon inscribed within collapsing star. Filled-in ellipses depict the hidden horizons at various spatial foliations.

a requirement, we work with isolated horizons which are characterized completely locally [17] within a non-stationary spacetime. These are depicted in collapsing situations in Fig. 2. The figure on the left shows two horizons forming in collapse, the formation of the first being followed by an accretion leading to the second. The figure on the right is the conformal depiction of the same spacetime. Since the characterization of isolated horizons is local, each of the horizons H and H' are equivalent isolated horizons with different cross-sectional areas and masses. In the language of event horizons, H would not qualify as an event horizon since it accretes matter; H' might be taken to be an event horizon provided no further accretion happens in future. But one needs to ‘know’ that in advance if one is to think of H' as an event horizon. One might expect to avoid this teleological conundrum by using *apparent* horizons, but then these latter are (a) described only on spatial foliations and (b) change discontinuously under physical changes like accretion. The second aspect stymies a description in terms of classical evolution in phase space which is continuous, and makes apparent horizons less suited for our intended formulation in terms of local variables. Isolated horizons afford such local descriptions. Briefly, they are characterized as follows :

- Isolated horizons are **Nonstationary**, i.e., are null boundaries of spacetimes which are not required to have global timelike isometries.
- An isolated horizon is a Null (lightlike) inner boundary of spm with topology $R \otimes S^2$.
- It is *marginally* Outer Trapped : in terms of the null geodesic generators l and n , $\theta_{(l)} = 0$, $\theta_{(n)} < 0$.
- An isolated horizon has cross-sectional area $\mathcal{A}(S^2) = \text{const} \rightarrow$ meaning of isolation.
- *Zeroth law of Isolated Horizon Mechanics* : surface gravity $\kappa_{IH} = \text{const}$ on isolated horizon, even though elsewhere it may not be possible to define this quantity due to lack of stationarity.
- It is possible to define *mass* on an isolated horizon : $M_{IH} = M_{IH}(A, Q)$ where, $M_{IH} \equiv M_{ADM} - \mathcal{E}_{rad}^\infty$, where M_{ADM} is the ADM mass, such that $\delta M_{IH} = \kappa \delta \mathcal{A}_{hor} + \Phi \delta Q_{hor}$ (*First law of Isolated Horizon Mechanics*)
- IH is *microcanonical ensemble* with fixed $\mathcal{A}_{hor}, Q_{hor}$
- Hawking radiation requires $IH \rightarrow$ Trapping or Dynamical Hor

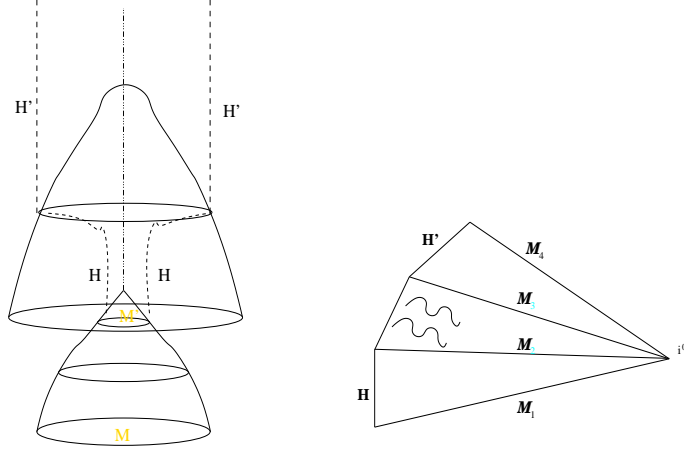


Figure 2: Isolated Horizons

The nascent, stable, hidden isolated horizon stage is followed by a stage of a hidden trapping/dynamical horizon [18, 17] which accretes energy (matter) and grows, and then settles down to a bigger hidden isolated horizon, still substantially inside the neutron star inner region. This isolated horizon changes to a trapping horizon and grows via accretion, subject to its fulfilling our stability criterion. This process of alternation between the hidden isolated and expanding trapping horizons continues until the entire star is engulfed by the isolated horizon which is then identified with *physically observed* horizon. The neutron star of course continues to shrink inside the horizon until it reaches the singularity. This proposed scenario is depicted in Fig. 4 in the Carter-Penrose frame.

3 Digression

3.1 Thermal Holography

In any version of quantum general relativity (QGR), the Hamiltonian constraint of the classical theory would be exhibited as the annihilation of bulk quantum states $|\psi_v\rangle$ (consisting of both the quantum bulk spacetime and matter states) by the bulk Hamiltonian operator, assuming that this latter exists,

$$\hat{H}_v |\psi_v\rangle = 0. \quad (3.1.1)$$

One can interpret (3.1.1) as the *quantum* Einstein equation. While classically, properties of the bulk spacetime determine completely properties of the boundary, in QGR this need not be the case. Thus, one may assume that the full Hilbert space $\mathcal{H} \sim \mathcal{H}_v \otimes \mathcal{H}_b$ where \mathcal{H}_b is the Hilbert space of quantum states describing boundary fluctuations. An arbitrary state in this Hilbert space may thus be expressed as

$$|\Psi\rangle = \sum_{v,b} c_{vb} \underbrace{|\psi_v\rangle}_{blk} \underbrace{|\chi_b\rangle}_{bdy}, \quad (3.1.2)$$

where we have assumed minimal entanglement between the bulk and the boundary, i.e., the coefficient matrix c_{vb} is not necessarily diagonal.

Consider now a canonical ensemble of such spacetimes in contact with a heat bath with an inverse

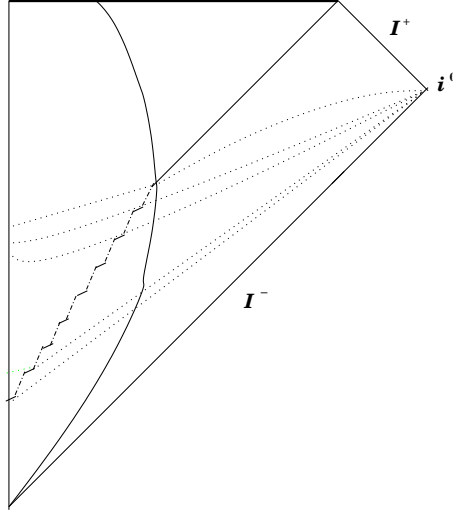


Figure 3: Proposed scenario for growth of hidden horizon in spherical collapse. Figure shows alternate isolated and trapping horizons culminating in emergence of physical isolated horizon. The part of the collapsing star to the left of the growing hidden horizon, at any spatial foliation (indicated by dotted lines) is to be taken to be *trapped*.

temperature β . The canonical partition function

$$\begin{aligned} Z &= \sum_b \left(\sum_v |c_{vb}|^2 ||\psi_v\rangle||^2 \right) \langle \chi_b | \exp -\beta \hat{H}_{bdy} | \chi_b \rangle \\ &\equiv Z_{bdy} , \end{aligned} \quad (3.1.3)$$

using the quantum Einstein equation for the bulk states. In other words, *the bulk states decouple* because of the quantum Einstein equation, and *thermodynamics (of any quantum spacetime) is completely described by the boundary states* [12, 14]! We term this phenomenon *thermal holography*; admittedly this is weaker result than the assertions in the *Holographic Hypothesis* [19, 20, 21], but a result nevertheless, albeit heuristic, rather than a conjecture.

3.2 Thermal Stability

Consider now the canonical ensemble above to consist of (inner) boundaries which are trapping or dynamical horizons which are thermally active. The result of the previous subsection, that the partition function is described in terms of the boundary states, can be interpreted, within the approach to quantum gravity known as Loop Quantum Gravity (LQG) [16], to imply that the quantum numbers labelling the states must correspond to some observable within LQG associated with such boundaries. One of the most important such observable is the area operator. Even though this operator is generically not a Dirac observable [16] in the strictest sense, the area operator corresponding to *physical* surfaces like (spatial foliations of) horizons are indeed good observables. In LQG, the spectrum of the area operator is discrete and bounded,

$$a_N = 8\pi\gamma A_P \sum_{p=1}^N \sqrt{j_p(j_p + 1)} \quad (3.2.1)$$

$$|a_N| \leq A_{hor} \pm O(A_P) , \quad (3.2.2)$$

where, the sum is over punctures on the horizon made by edges of spin network states describing quantum spacetime. Now, for macroscopic horizons whose areas $A_{hor} \gg A_P$, one can consider for simplicity, states where all spins at punctures are $1/2$. In this situation, $a_N \sim N A_P$, $N \gg 1$.

Continuing with the canonical ensemble considered in the previous subsection, the boundary partition function corresponding to a canonical ensemble of thermally active trapping horizons can be reexpressed in terms of functions of the area eigenvalues a_N [22, 13, 14, 15]

$$Z_b(\beta) = \sum_n g(M(a_n)) \exp -\beta M(a_n) , \quad (3.2.3)$$

where, we have introduced an unspecified function $M(a_N)$ of the horizon area eigenvalues as the *mass* of the horizon. We now assume that the ensemble has an equilibrium configuration which is an *isolated* horizon as defined in section 2. We also assume that the partition function (3.2.3) can be evaluated in saddle point approximation including Gaussian fluctuations around the saddle point, taken to be an isolated horizon of cross-sectional area A_{IH} . Changing the sum to an integral for large horizon areas ($N \gg 1$), we obtain [22, 13, 14]

$$Z_b(\beta) \simeq \exp [S(A_{IH}) - \beta M(A_{IH})] \cdot \Delta^{-1/2}(A_{IH}) , \quad (3.2.4)$$

where, $S(A_{IH}) \equiv \log g(A_{IH})$ is the *microcanonical* entropy of the equilibrium isolated horizon, and

$$\Delta(A_{IH}) \equiv M_A^{-1} [M_{AA} S_A - S_{AA} M_A] , \quad (3.2.5)$$

where the subscripts A imply derivatives of these functions with respect to their argument and we have dropped the subscript IH from the A_{IH} for convenience.

Evaluating the *canonical* entropy using standard thermodynamic formulae, we get

$$S_{canon}(A) = S(A) + \frac{1}{2} \log \Delta(A) \quad (3.2.6)$$

where, as already mentioned, $S(A)$ is the microcanonical entropy of the equilibrium horizon taken as the saddle point, we see that, for S_{canon} to have the standard interpretation of entropy, we must have $\Delta(A) > 0$. Integrating the corresponding differential inequality, using (3.2.5), and reinserting the fundamental constants set to unity earlier, we get the condition

$$\frac{M(A_{IH})}{M_P} > \frac{S(A_{IH})}{k_B} . \quad (3.2.7)$$

We take this condition to be the *necessary and sufficient condition* for the equilibrium isolated horizon IH to be thermally stable [23, 14]. If this inequality is violated, the isolated horizon is not a stable equilibrium configuration and the resulting instability leads to an incessantly radiating (or thermally accreting) trapping horizon, which may eventually evaporate and disappear (in the case of radiation). A similar thermal stability criterion has been derived recently for *charged* trapping horizons [15].

The stability criterion (4.0.6) depicts the domination or otherwise of processes that are ‘energy (mass)-driven’ rather than ‘entropy-driven’. In other words, in the context of the collapsing neutron star, the domination of entropy-driven processes may be considered tantamount to the domination of Fermi pressure due to Pauli exclusion in the neutron core, which may well stave-off gravitational collapse to a black hole. The concept of a hydrostatic equilibrium between ‘forces’ due to gravity and that due to Fermi pressure of degenerate neutrons is replaced here by an ‘equilibrium’ between the horizon mass and its microcanonical entropy - both statistical concepts rooted in quantum aspects of spacetime.

3.3 Microcanonical Entropy

One observes that the non-triviality of the stability criterion (4.0.6) is crucially dependent on the microcanonical entropy being a more general function of the horizon area than merely linear (i.e., merely

Bekenstein-Hawking). Indeed, the Bekenstein-Hawking area law for black hole entropy was historically derived using *semiclassical* arguments where a classical spacetime metric is used. It is thus subject to modifications within a genuine proposal for *quantum* spacetime geometry like LQG. Indeed, longer than a decade ago, the microcanonical entropy of spherical isolated horizons was first investigated [24, 25, 26, 27], on the basis of counting states of an $SU(2)$ Chern Simons theory describing the dynamics on the horizon, as derived using LQG. Using the connection [28] between the Chern Simons theory Hilbert space on a 3-manifold and the conformal current blocks of the Wess-Zumino-Witten model on the boundary of the 3-manifold, the microcanonical entropy was shown, for macroscopic ($A_{IH} \gg A_P$) horizons, to be given by an infinite series, asymptotic in the horizon area [26, 29], beginning with the BH area law, but with calculable and finite corrections :

$$S(A_{IH}) = S_{BH}(A_{IH}) - \frac{3}{2} \log S_{BH}(A_{IH}) + \text{const} + O((S_{BH}^{-1})) , \quad (3.3.1)$$

where, $S_{BH}(A_{IH}) \equiv A_{IH}/4A_P$, $k_B = 1$ is the BH area law for entropy. Because of the logarithmic corrections (and beyond) in (3.3.1), S_{AA} is nontrivial, leading to the form of the stability criterion given in (4.0.6). This underlines the essentially quantum gravitational nature of thermal stability of black holes, since, instead of using the classical metric and properties derived from classical geometry, LQG aspects have been germane to the derivation of (4.0.6).

4 Critical Mass

Assume that the fractional energy (mass) loss of the collapsing neutron star is small, such that the original mass of the star is more or less completely engulfed by the horizon,

$$\frac{M_{crit}}{M_P} > \xi \frac{M_{IH}}{M_P} . \quad (4.0.1)$$

Recall our picture on the right panel of Fig. 1 where, a nascent ‘embryonic’ quasi-isolated horizon which is stable according to our criterion (4.0.6), changes to a trapping horizon TH_0 which accretes neutron-rich material from its environment and grows. Thus, one has a series of alternate (quasi-) isolated and trapping horizons in the sequence, pictorially depicted in Fig. 3.

$$IH_0 \rightarrow TH_1 \rightarrow IH_1 \rightarrow TH_2 \rightarrow \dots TH_N \rightarrow IH \text{ (physical)} . \quad (4.0.2)$$

This sequence requires the stability conditions for the quasi-isolated horizons IH_0, \dots, IH_N

$$\frac{M_{IH_i}}{M_P} > \frac{S_{IH_i}}{k_B} , \quad i = 0, 1, 2, \dots, N \quad (4.0.3)$$

implying the sequence of inequalities

$$A_{IH_0} < A_{IH_1} < A_{IH_2} < \dots < A_{IH} \text{ (physical)} . \quad (4.0.4)$$

Observe that if any of the inequalities (4.0.3) fails to hold, this implies that the hidden quasi-isolated horizon is unstable, and will no longer change to a trapping horizon which will accrete and grow. Rather, the nucleation process stops there and the hidden horizon simply dissolves into the neutron star core.

If indeed, as assumed, IH_0 is the incipient quasi-isolated horizon, then A_{IH_1} is the cross-sectional area of the isolated horizon which the trapping horizon TH_1 has settled into. In this case, since the foliation of TH_1 is an outer trapping surface, $A_{IH_1} > A_{Cn}$. The sequence of inequalities (4.0.4) then implies

$$\frac{A_{IH}}{A_P} > \frac{A_{Cn}}{A_P} . \quad (4.0.5)$$

Recalling the stability criterion applied now to the *actual* isolated horizon

$$\frac{M(A_{IH})}{M_P} > \frac{S(A_{IH})}{k_B} = \xi \frac{A_{IH}}{A_P}, \quad (4.0.6)$$

and using eq.s (4.0.5) and (4.0.1), we obtain

$$\frac{M_{crit}}{M_P} > \xi \frac{A_{Cn}}{A_P} \quad (4.0.7)$$

from which eq. (1.0.1) follows immediately. Thus, upto the $O(1)$ constant ξ , the minimum mass of a collapsing neutron star for which it will collapse to a black hole, obtained from the above analysis, has the correct order of magnitude in terms of fundamental constants (or the solar mass).

5 Discussion

The formulation of the problem of critical mass of a neutron star beyond which it becomes unstable with respect to gravitational collapse to a black hole, has been made here in a *dual* fashion, namely, by investigating the *minimum* mass for which such a gravitational collapse occurs, with the concomitant formation of an event (isolated) horizon. Thus, the focus has been on the description of horizon formation rather than the instabilities of a neutron star brought about by strong interaction dynamics of a dense neutron gas in its interior, in a background of strong gravity. This shift of focus has enabled the facile use of results pertaining to horizon dynamics and thermodynamics which result from various aspects of quantum gravity, especially LQG. The complications associated with the traditional approach regarding possible inconsistencies in using classical *general* relativity to determine the hydrostatic pressure in the neutron star interior, while using an equation of state determined from *special* relativistic quantum field theory, have been thus completely obviated. Further, this formulation underlines the *quantum* gravitational underpinnings of the process of horizon formation, in case gravitational collapse does occur. This is a novel feature of our assay in comparison with that of ref. [8] where the incipient predominance of gravitational effects over the strong dynamics of densely packed neutrons is already in evidence. However, it is not possible to obtain a thermal stability criterion like we have derived, on the basis of classical general relativity.

Indeed, horizon thermodynamics is inherently quantum to the extent that quantum gravity affords the only sensible ab initio description of notions like black hole entropy [12, 11]. Even the entropy of galactic centre black holes weighing billions of solar masses has its origin in quantum gravity, i.e., microstates describing physics at 10^{-33} cm. Black holes must therefore be an extreme example of macroscopic quantum phenomena. While the entropy of isolated black holes is now reasonably well-understood on the basis of quantum gravity (e.g., LQG), a myriad aspects of quantum black holes are yet to be understood in detail, chief among which is the proper understanding of black hole singularities. This aspect is of crucial importance if one is to resolve the so-called Information Loss Paradox. On our part, the thermal stability of black holes has been a subject of earlier investigations, and our result from that study appears to yield the critical mass more or less straightforwardly. Nevertheless, there are features of our analysis which are heuristic in character, and these features need thorough reexamination before any claim of rigour can be made.

Neutron stars are currently detected as sources of moderately hard X-rays, with tell-tale accretion discs in case they are a partner in an X-ray binary system. It is not easy at all to decipher from X-ray data (or even from radio frequency data emitted by pulsars) information regarding the degrees of freedom and dynamics in the interior of neutron stars. The somewhat unexpected link seen here between the critical mass and the entropy of the nascent hidden horizon emerges from a proposed scenario of horizon formation by a process of ‘bubble’ nucleation as in a first order phase transition. At this stage of development, it

is far from clear what would constitute an observational evidence for this phase transition. There are models of neutron star structure where certain layers close to the outer boundary may have superfluid or superconducting properties, and there are expectations that a careful study of the X-ray spectrum may yield observational evidence for this. In this vein, it is possible that the hidden horizon growth discussed in the text of the paper may also radically influence the X-ray spectrum towards the later stages of the collapse.

Finally, we note in passing that in the classical description of gravitational collapse of spherical pressureless dust by J. R. Oppenheimer and H. Snyder, or that of a spherical distribution of electromagnetic radiation by P. C. Vaidya, the growth of a trapped surface deep inside the collapsing matter or radiation may actually be studied as toy models within classical general relativity. We hope to report on this in the near future [30].

Acknowledgement: I thank D. Bandyopadhyay for illuminating discussions and for guiding me to the literature on neutron stars.

References

- [1] E. Massot, J. Margueron and G. Chanfray, *Europhys. Lett.* **97**, 39002, 2012; e-Print: arXiv:1201.2772 [nucl-th].
- [2] H.-J. Schulze, T. Rijken, *Phys.Rev.* **C84**, 035801, 2011.
- [3] T. Yazdizadeh and G. H. Bordbar, *Res. Astron. Astrophys.* **11** 471, 2011; e-Print: arXiv:1012.2939 [astro-ph.SR]
- [4] I. Vidana, D. Logoteta, C. Providencia, A. Polls, I. Bombaci, *Europhys. Lett.* **94** 11002, 2011; e-Print: arXiv:1006.5660 [nucl-th].
- [5] H.-J. Schulze, A. Polls, A. Ramos and I. Vidana, *Phys. Rev.* **C73**, 058801, 2006.
- [6] G-Q Li, C.H. Lee, G.E. Brown, *Phys. Rev. Lett.*, **79**, 5214, 1997. e-Print: nucl-th/9711002
- [7] M. Prakash, T. L. Ainsworth and J. M. Lattimer, *Phys. Rev. Lett.* **61**, 2518, 1988.
- [8] C. E. Rhoades and R. Ruffini, *Phys. Rev. Lett.*, **32**, 324, 1974.
- [9] S. Chandrasekhar, *Astrophys. Jou.* **74**, 81, 1931. See also, S. Chandrasekhar, *Nobel Lecture (Physics) 1983 in Nobel Lectures, Physics 1981-1990*, Chief Ed. T. Frängsmyr, Editor G. Ekspang, World Scientific Publishing Co., Singapore, 1993.
- [10] R. M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*, Chicago, 1995.
- [11] R. K. Kaul, *SIGMA* **8**, 005, 2012. e-Print: 1201.6102 [gr-qc].
- [12] P. Majumdar, *Int. Jou. Mod. Phys.* **A24**, 3414, 2009. e-Print: arXiv:0903.5080 [gr-qc].
- [13] A. Chatterjee and P. Majumdar, *Phys. Rev. Lett.* **92**, 141302, 2004. e-Print: gr-qc/0309926.
- [14] P. Majumdar, *Class. Quant. Grav.* **24**, 1727, 2007. e-Print: gr-qc/0701014.
- [15] A. Majhi and P. Majumdar, *Charged Quantum Black Holes: Thermal Stability Criterion*, e-Print: arXiv: 1108.4670 [gr-qc].
- [16] C. Rovelli, *Quantum Gravity*, Cambridge, 2004.

- [17] A. Ashtekar and B. Krishnan, *Liv. Rev. Rel.* **7**, 10, 2004.
- [18] S. A. Hayward, *Phys. Rev.* **D49**, 6467, 1994.
- [19] G. 't Hooft, *Dimensional Reduction and Quantum Gravity*, in *Salamfestschrift*, edited by A. Alo and S. Randjbar-Daemi, ICTP, Trieste, 1993.
- [20] L. Susskind, *Jou. Math. Phys.* **36**, 6377, 1995. e-Print: hep-th/9410074.
- [21] R. Bousso, *Rev. Mod. Phys.* **74**, 825, 2002. e-Print: hep-th/0203101.
- [22] S. Das, P. Majumdar, R. K. Bhaduri, *Class. Quant. Grav.* **19**, 2355, 2002. e-Print: hep-th/0111001,
- [23] A. Chatterjee and P. Majumdar, *Phys. Rev.* **D72** 044005, 2005. e-Print: gr-qc/0504064.
- [24] A. Ashtekar, J. Baez, A. Corichi and K. Krasnov, *Phys. Rev. Lett.* **80**, 904, 1998. e-Print: gr-qc/9710007.
- [25] R. K. Kaul and P. Majumdar, *Phys. Lett.* **B439**, 267, 1998. e-Print: gr-qc/9801080.
- [26] R. K. Kaul and P. Majumdar, *Phys. Rev. Lett.* **84**, 5255, 2000. e-Print: gr-qc/0002040.
- [27] A. Ashtekar, J. Baez and K. Krasnov, *Adv. Theor. Math. Phys.* **4**, 1, 2000. e-Print: gr-qc/0005126.
- [28] E. Witten, *Commu. Math. Phys.* **121**, 251, 1989.
- [29] S. Das, R. K. Kaul, P. Majumdar, *Phys. Rev.* **D63**, 044019, 2001. e-Print: hep-th/0006211.
- [30] C. Chakraborty and P. Majumdar, in preparation.